A Network Flow Method for Determining the Gradient of Roads

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The paper describes a conversion of the road “cut and fill” design problem for solution by a network algorithm.

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1. INTRODUCTION
There is a class of mathematical programming models which is so very much simpler and faster to solve than even the general linear programming problem that it sometimes warrants some distortion of the real-world problem to make it fit. This class is the totally unimodular linear programming problems, of which the most familiar examples are the so-called Transportation problem and the Assignment problem. This paper is to draw attention to another example, which arises in the field of road design – in particular for a network of residential roads.

Once the plan of the road network has been determined, a second design stage is to determine the vertical profiles of the roads. Since there are constraints on the profiles, and costs associated with cutting and filling, it is not surprising that a mathematical programming model should be applicable. Under the following assumptions it is possible to have a unimodular formulation.

1.1 Assumptions
1. The plan of the road system has been decided and only the vertical profiles remain to be determined.
2. Only the gradients of the roads are subject to constraint, not curvatures (second derivative of vertical position). Other constraints take the forms of limitations on height differences, or specified heights, only.
3. An adequate representation of the ground contours can be given by a limited grid of points in the plane.
4. Cost of construction can be adequately approximated by a linear function (or piecewise linear convex function) of the height of the road above or below the ground surface at each grid point.
5. The gradient constraints can be adequately represented by maximum absolute differences of the road between (normally, but not necessarily, geographically adjacent) grid points.